Problem Set 2
Inequalities

(1)

If a, b, c, and d are positive numbers such that $a + 2b + 3c + 4d = 8$, then what is the maximum value of the product $abcd$?

(2)

If x and y and real and $x^2 + y^2 = 1$, compute the maximum value of $(x + y)^2$.

(3)

Find the minimum value of the function $g(x, y, z) = \frac{x}{y} + \sqrt{\frac{y}{z}} + \sqrt[3]{\frac{z}{x}}$.

(4)

What is the smallest positive integer n such that $\sqrt{n} - \sqrt{n-1} < .01$?

Hint: What does the decimal suggest we do?

(5)

I travel to my destination a distance k away with a velocity of v_1. I travel back the same distance k with a velocity v_2. Express my average speed in terms of v_1 and v_2 (If you already know it, prove it). What kind of mean is this?

(The answer is not $\frac{v_1+v_2}{2}$, arithmetic)

Challenge Questions

(1)

Prove that $\sin^2(\theta) \geq 0$ only holds for real θ.

Hint: Remember Euler's Formula.

(2)

Find the smallest integer n such that

$$(x^2 + y^2 + z^2)^2 \leq n(x^4 + y^4 + z^4)$$

Holds for real x, y, and z.

(3)

If a, b, and c are each positive and $a + b + c = 6$, show that

$$\left(a + \frac{1}{b}\right)^2 + \left(b + \frac{1}{c}\right)^2 + \left(c + \frac{1}{a}\right)^2 \geq \frac{75}{4}$$

Hint: Try to construct this inequality from what we know.