

Newport Math Club

BASES

Knowledge Check

- ⦿ If you want to prove you know everything (well, just the stuff we are covering today):
- ⦿ Convert 221_3 to hex...
 - No Talking
 - No Sharing

Counting

- ① Start counting. This may seem like a basic skill you learned a long time ago, but we're going to look at it a little differently
- ② What happens after 9?
- ③ What happens after 99?
- ④ Do you see a pattern?

Anatomy of a Number

- Let's consider carefully the number 1234.
- We're going to label each digit, starting with 0 on the right.

1	2	3	4
3	2	1	0

Anatomy of a Number

- ⦿ What does the 4 mean? You may have learned a long time ago that it is in the units digit.
- ⦿ What does the 3 mean? We have three 10s.
- ⦿ The 2? We have two 100s.

1	2	3	4
3	2	1	0

Anatomy of a Number

- Our number system is the decimal system, meaning that we work in base 10.

1	2	3	4
$\times 10^3$	$\times 10^2$	$\times 10^1$	$\times 10^0$
= 1000	= 200	= 30	= 4

- Wow! The sum of the resulting values is 1234.

Octal

- Octal is the name for base 8. It is very similar to decimal.
- In base 10, we have digits called 0 – 9. Once we hit 10, we start repeating digits.
- In base 8, we have digits 0 – 7. Once we hit 8, our number has two digits.
- Count to 10 in octal! 1, 2, 3, 4, 5, 6, 7, 10!
- ... 11, 12, 13, 14, 15, 16, 17, 20... 76, 77, 100

From Octal to Decimal

- ⦿ A base is often denoted with a subscript.
- ⦿ Let's consider the octal number 514_8 .

5	1	4
$\times 8^2$	$\times 8^1$	$\times 8^0$
$= 320$	$= 8$	$= 4$

- ⦿ If we add those up, we get 332. This means that 514_8 is equal to 332_{10} !

From Decimal to Octal

- ⦿ Let's say we have the decimal number 729.
- ⦿ How do we convert it to octal? First, let's figure out all the places in octal...

512	64	8	1

From Decimal to Octal

512	64	8	1
1	3	3	1

- ⦿ In 729, there is one 512, so we put a 1 in the 512s place. This leaves $729 - 512 = 217$
- ⦿ In 217, there are three 64s, so we put a 3 in the 64s place. This leaves $217 - 3(64) = 25$
- ⦿ We are left with three 8s and a 1.
- ⦿ Now we have converted 729_{10} to 1331_8 !

Other Common Bases

- ◎ Binary: Base 2 (ex. 101101100010101)
 - Count: 1, 10, 11, 100, 101, 110, 111, 1000, 1001...
 - Used in computers, since each digit is “on” or “off”
- ◎ Hexadecimal (Hex): Base 16 (ex. 512A7F4)
 - The letters A-F are used to denote digits over 9
 - Count: 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11...
 - Used in colors, among other things (FFFFFF is white)

Your Turn

- ⦿ Convert 152_8 to base 10.
- ⦿ Convert 425_6 to base 10.
- ⦿ Convert 963_{10} to hexadecimal.
- ⦿ Convert 66_{10} to binary.
- ⦿ Convert 215_7 to base 9.
- ⦿ Convert 10110001_2 to base 4.
- ⦿ Challenge! Convert 156_{10} to base -2.

Negative Bases

- ⦿ Yes, they sound hard, but they aren't that bad. What really is a negative base?
- ⦿ Convert 211_{-3} to base 10...

2	1	1
$\times (-3)^2$ or 9	$\times (-3)^1$ or -3	$\times (-3)^0$ or 1
= 18	= -3	= 1

- ⦿ Add them up, and we get 16_{10} .

Negative Base Practice

- ⦿ Multiply -103_{-4} and 312_{-4} in base -4 .
 - A) Express your answer as a positive number.
 - B) Express your answer as a negative number.

Questions?