Newport Math Club

NUMBER THEORY

Part 1

3
5
9
1
2
4
5
7
9
8
1
6
4
3
2
1
8
7
5
1
6
9
4

What is a Prime Number?

- A prime number is any positive integer that is not divisible by any other positive integer except 1 and itself
- 1 is **not** a prime number.
- The first prime numbers are:
  - 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37…
- Quick! Is 323 prime?
  - Answer: NO! It is divisible by 17 and 19

What is a Factor?

- A factor is any number or term that multiplies with another
  - ±1, ±2, ±3, ±6 are factors of 6.
  - (x + 3) and (x – 1) are factors of $x^2 + 2x – 3$
- Technically, a number can have infinitely many factors
- Most of the time, we want to talk about integer (whole number) factors, and often only the positive ones

Prime Factors

- A prime factor is a factor of a number that is a prime number
- Therefore, prime factors are always integers greater than 1
- If you multiply all of the prime factors of a number together, you get the number
- Prime factors create a sort of unique “signature” for every integer above 1

How to Find Prime Factors

- Let's consider the number 720
- There are two ways to find the prime factors – but both methods involve dividing the number into smaller parts until you reach 1

Table Method

- List the prime factors on the right and the quotients on the left:

720	Prime factors
360	2
180	2
90	2
45	2
15	3
5	3
1	5

As you can see, we divide our number by prime numbers until we reach 1 on the lower left.

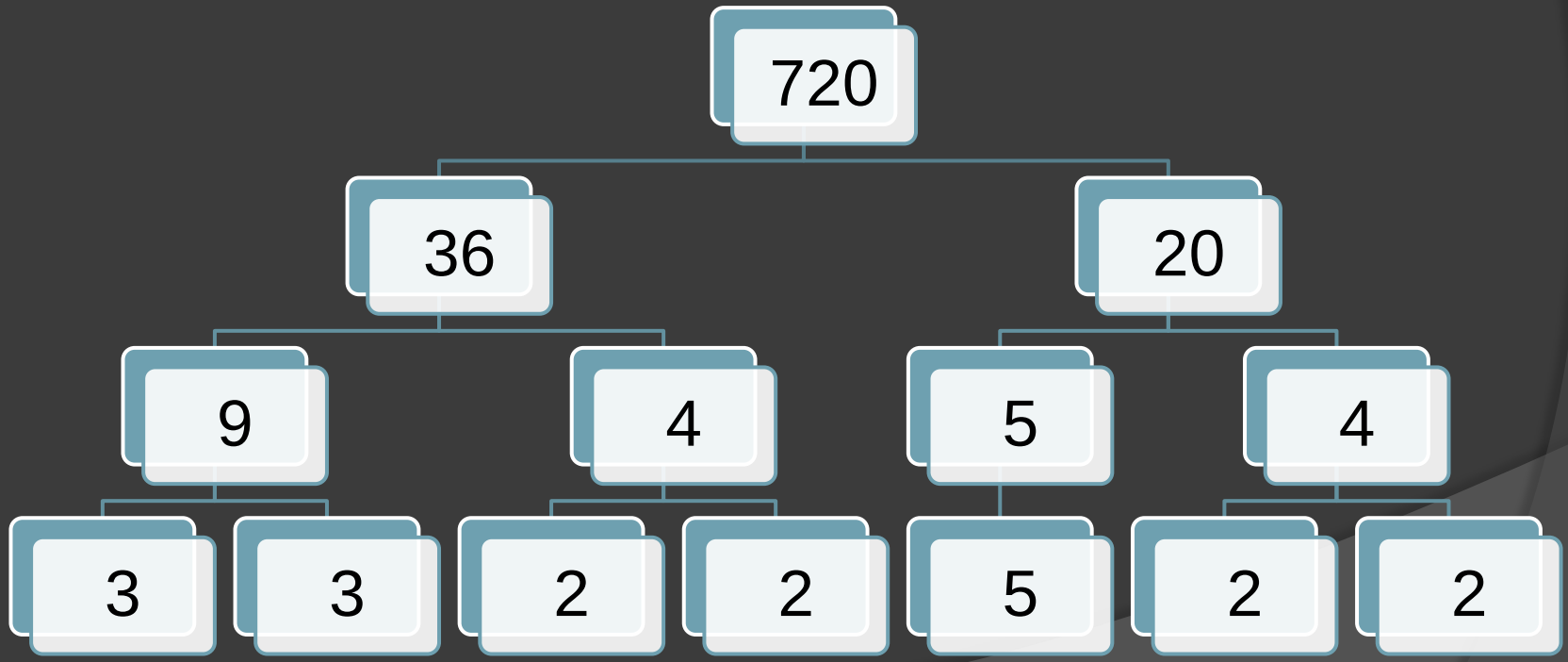
Prime Factorization

- Now, we have our prime factorization:
- 2 x 2 x 2 x 2 x 3 x 3 x 5
- There is no way to break this down any smaller
- We can verify that the above factors indeed multiply to 720
- We often express the prime factorization using exponents for factors with *multiplicities* > 1:
  $2^4 \times 3^2 \times 5$ ← Prime Factorization!

Prime factors
2
2
2
2
3
3
5

Tree Method

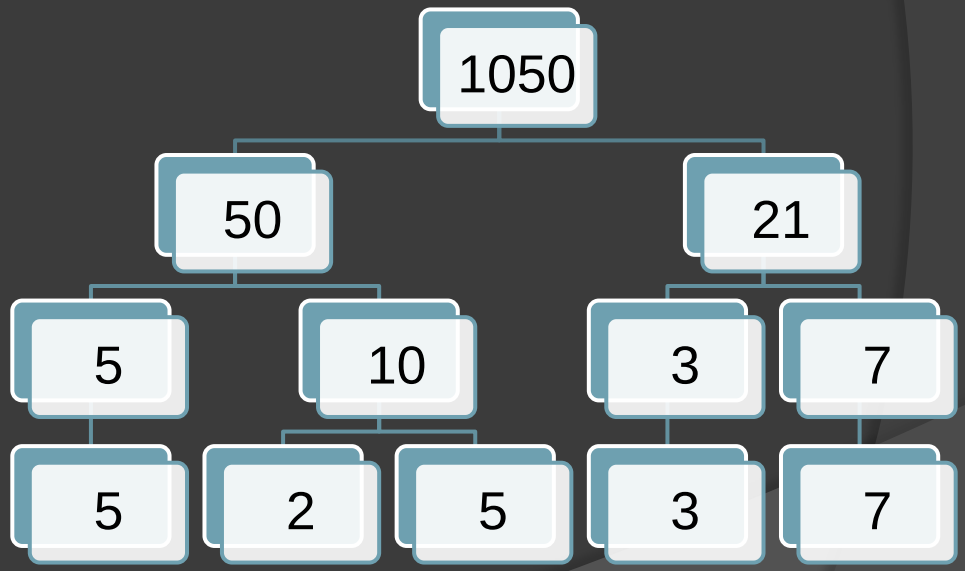
- This method is often a faster way to find the prime factorization of a number.



Your Turn

Let's consider two numbers: 720 and 1050
Find the prime factorization of 1050

1050	Prime Factors
525	2
175	3
35	5
7	5
1	7



Greatest Common Divisor (GCD)

- Find the common prime factors:

720:	2	2	2	2	3	3	5		
1050:	2				3		5	5	7

- We now know that the greatest common divisor, the largest integer that divides evenly into both numbers (750 and 1050), is 2 x 3 x 5, or 30
- If there are no factors in common, the GCD is 1 and the numbers are *relatively prime*

Least Common Multiple (LCM)

- Make a prime factor chart, just like for GCD

720:	2	2	2	2	3	3	5		
1050:	~~2~~				~~3~~		~~5~~	5	7

- Cross out the duplicates
- Multiply the remaining numbers:
  - 2 * 2 * 2 * 2 * 3 * 3 * 5 * 5 * 7 = 25200
- 25200 is the LCM, the smallest number that is divisible by both 720 and 1050.
- Think of it as finding a *common denominator*

Some quick notes...

- Which is greater, the GCD or the LCM?
- Are they ever the same?

- Use common sense:
  - Quick! Find the prime factorization of 169
  - Quick! Find the GCD of 144, 432, and 7
  - Quick! Find the LCM of 8, 16, and 32

Answers: 13^2, 1, 32

Your Turn

- How many *distinct* prime factors does 8192 have? 1
- What is the prime factorization of 480? $2^5 \cdot 3 \cdot 5$
- What is the GCD of 686 and 294? 98
- What is the LCM of 42 and 60? 420
- What is the GCD of 729 and 512? 1
- What is the LCM of 100 and 147? 14700
- What is the LCM of 60, 45, and 77? 13860

Continued in Part 2…